Modal Parameter Estimation for Large, Complicated MIMO Tests

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Multiple-input, multiple-output (MIMO) experimental modal testing is often used for large structures. The data collected are used in multiple-reference reduction schemes to find the best set of modal parameters to describe the system. Often several or many of the reference shakers do not adequately excite all of the modes from each reference location. When this is the case, using all of the reference data may produce modes that are not optimum. A careful selection of references for generating modal parameters is critical for developing a good modal database for design, analysis, simulation and correlation efforts. While this is true of earlier modal parameter estimation algorithms, the latest PolyMAX estimation algorithm has significant advantages over historically used techniques.

Experimental modal tests are often conducted using a multiple–input, multiple-output testing strategy. Depending on the complexity of the structure to be tested, two or more shakers may be used for the excitation of the system. Many times it is very difficult, if not impossible, to have all the shakers excite all the modes of the system equally. This is especially true when the structure exhibits directional global modes or when the structure has an abundance of local modes due to appendage or subcomponent modal energy. When this is the case,

Figure 1. RADARSAT1 under test.

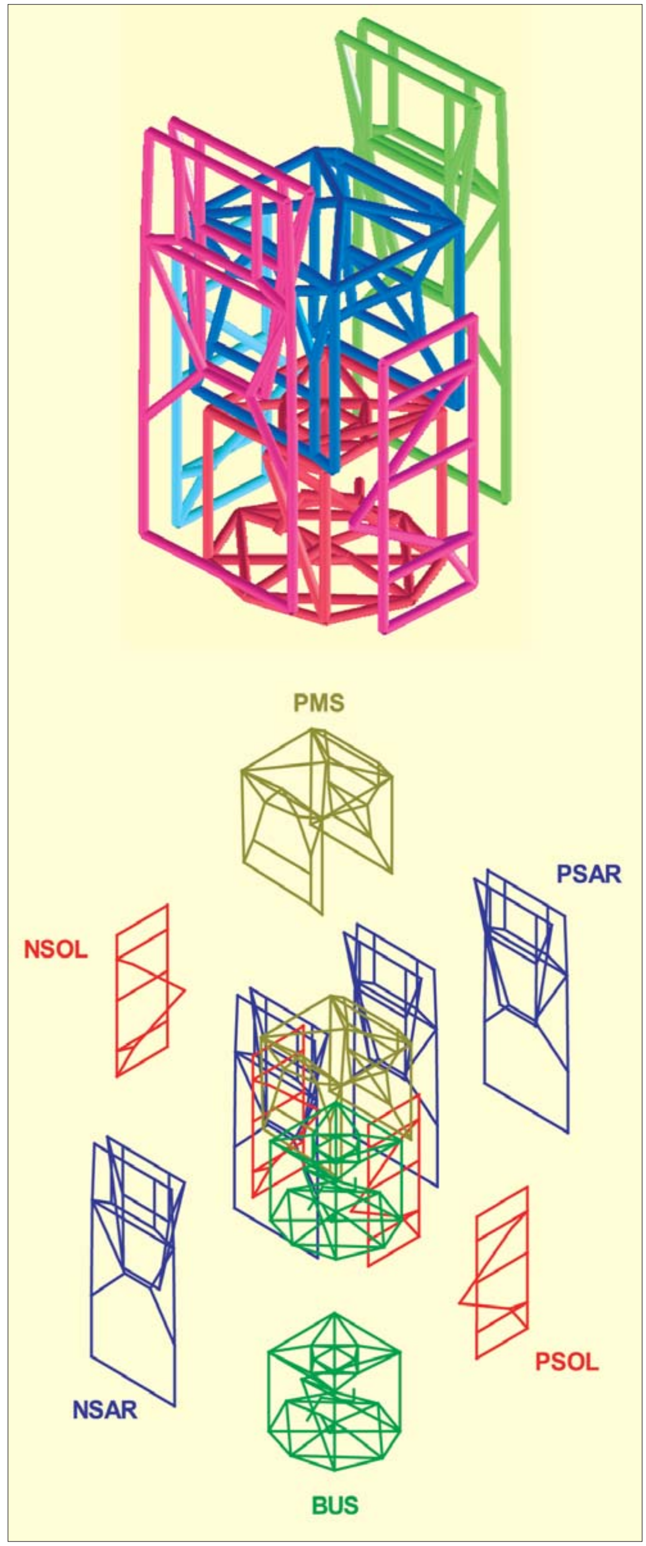

Figure 2. RADARSAT1 test geometry.

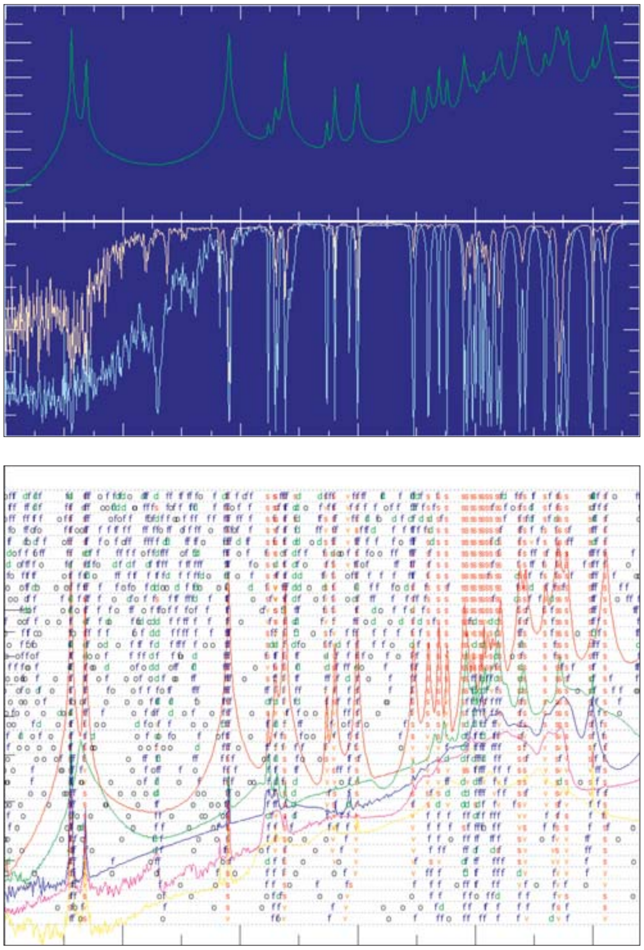

Figure 3. Summation function and mode indicator function (top) using all references along with stability diagram (bottom) over entire 10-64 Hz band.

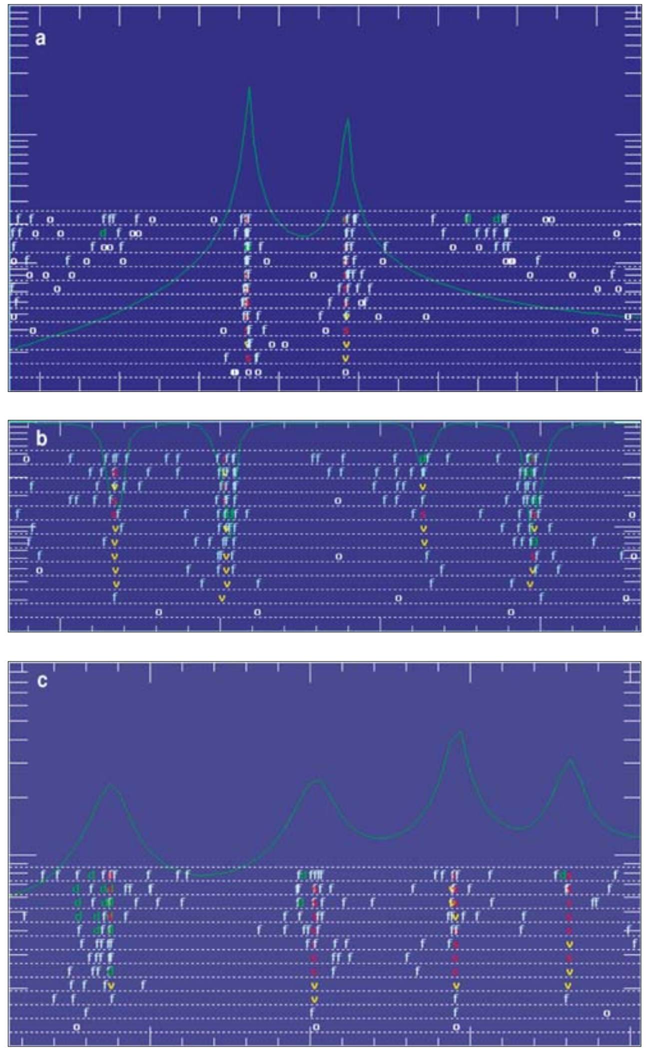

Figure 4. Stability diagram for three different bands using all references: a) 12.6-20.6 Hz; b) 36.7-40.6 Hz; and c) 44.1-48.1 Hz.

multiple-shaker testing is necessary to adequately excite all the modes of the system over the desired frequency range.

However, all of the shakers may not exhibit a high degree of modal participation for each individual mode of the system. In this case, the extraction of modal parameters may be affected by the inability to adequately excite every mode to a sufficient degree. If this is the case, the modal participation will reflect this and the resulting modal parameters are weighted by the modal participation values. This is handled in the extraction phase of the modal parameter estimation process. However, there is a serious concern when modal participations are below 20% and especially if they are below 10% of the total participation of the other shakers exciting the system. When these participations drop to such low values, the modes of the system are not adequately excited, or excitation directions and resulting measurements are generally not particularly good. The coherence of these measurements that are not well excited is also affected and is generally not very good. The measurements then are not considered optimum. The main problem is that the measurements contaminate the overall extraction of modal parameters from good reference locations using traditional approaches.

To extract the best possible modal parameters, it may be necessary to exclude certain measurements that are not considered particularly good from the global modal parameter pole extraction. Using *all* of the measured data may not produce the best overall extracted modal parameters. A careful review of all the measurements and modal participation factors may help to determine the best set to use in extracting the best modal parameters to describe the system.

However, more recent advances in modal parameter estimation have yielded new processing algorithms that are not as sensitive to the requirements identified above. PolyMAX[1,2] is a newer algorithm that can overcome many of the limitations identified above. Wide frequency bands with all measurement degrees of freedom can be effectively processed with little numerical or user difficulty.

To illustrate some of the problems associated with using a complete set of multiple-input, multiple-output frequency response measurements using more traditional modal parameter estimation techniques, several modal parameter estimation scenarios are explored. One extraction uses all of the measured data, and the other uses a selected set of frequency response measurements to show differences that exist in the extraction process. Both of these utilize older modal parameter estimation technologies common in almost all software packages available today.[3] In addition, the same data set was also processed using the new PolyMAX[4] approach to illustrate the differences and advantages of this new technique.

To illustrate the concerns in processing data, the Canadian RADARSAT1 satellite experimental modal test shown in Figure 1 was used for demonstration purposes. This experimental modal test was conducted with several shaker excitations applied to the structure. The modal test consisted of 250 response accelerometers resulting from five separate shaker excitation locations. Upon reduction of the data, the lower 25 modes of the system can be seen to be most directly excited by only two of the shaker excitation locations. Reduction of the data was performed using all of the shaker excitation locations. A more selective set of excitation locations was used based on the modal participation factors for each of the modes of the system to illustrate the degradation of the modal data when using all data references simultaneously. The data set was then finally

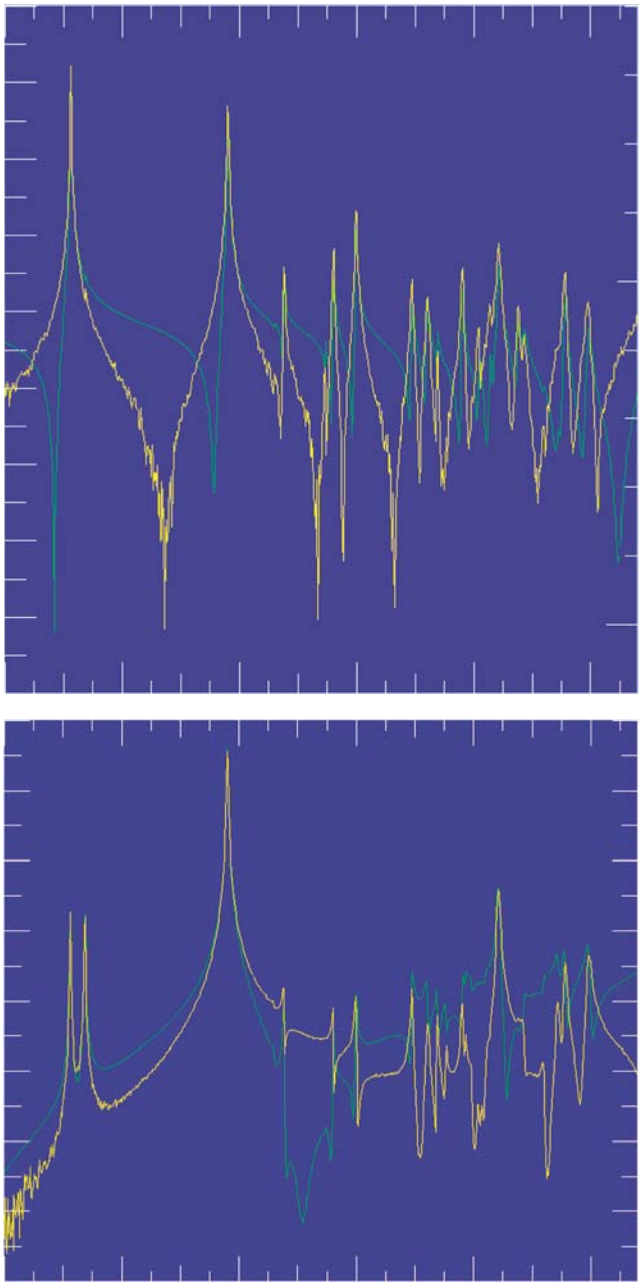

Figure 5. Typical synthesized FRFs using all references, 10-64 Hz.

processed using the PolyMAX technique. First, the data were reduced using traditional approaches to show the amount of effort and manipulation required to extract useful modal parameters. Then, the data were processed using the newly developed PolyMAX technique.

Theory

The extraction of modal parameters involves several basic equations related to modal analysis theory. These equations are briefly summarized to show the effects of different reference locations on the extraction of modal parameters.

Frequency Response Measurement Formulation. The frequency response function can be expressed in terms of the summation of the modes of the system. One form of this equation represents the modal characteristics of poles and residues as:

$$\left[H(s)\right]_{s=j\omega} = \left[H(j\omega)\right] = \sum_{k=1}^{m} \frac{\left[A_k\right]}{(j\omega - p_k)} + \frac{\left[A_k^*\right]}{\left(j\omega - p_k^*\right)} \quad (1)$$

For a particular mode k the frequency response can also be shown to be expressed as the singular valued decomposition of the system matrix as:

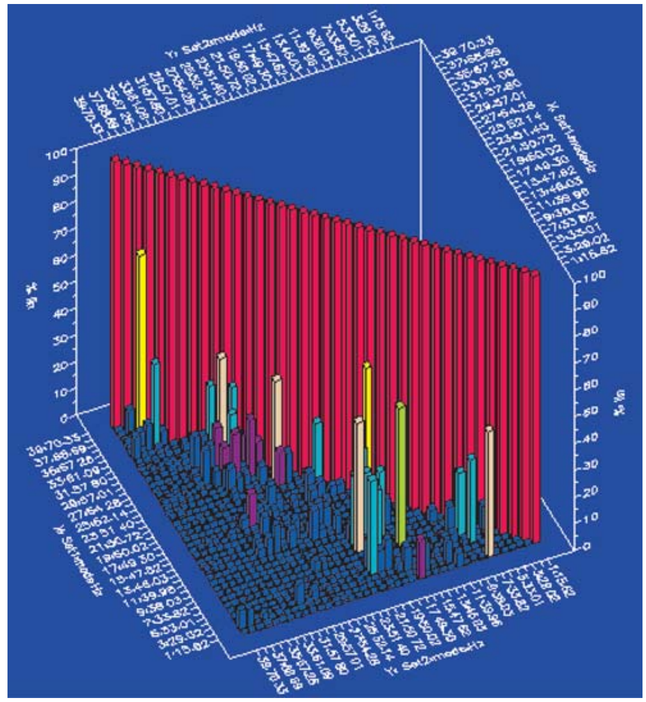

Figure 6. MAC of modes using all references.

$$\left[H(s)\right]_{s=p_k} = \left\{u_k\right\} \frac{q_k}{s - p_k} \left\{u_k\right\}^T \quad (2)$$

In this formulation, the residue matrix is therefore related to the mode shapes in the classical representation as:

$$\left[A(s)\right]_k = q_k \left\{u_k\right\} \left\{u_k\right\}^T \quad (3)$$

Upon expanding some of the terms of this expression,

$$\begin{bmatrix} a_{11k} & a_{12k} & a_{13k} & \cdots \\ a_{21k} & a_{22k} & a_{23k} & \cdots \\ a_{31k} & a_{32k} & a_{33k} & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix} = q_k \begin{bmatrix} u_{1k}u_{1k} & u_{1k}u_{2k} & u_{1k}u_{3k} & \cdots \\ u_{2k}u_{1k} & u_{2k}u_{2k} & u_{2k}u_{3k} & \cdots \\ u_{3k}u_{1k} & u_{3k}u_{2k} & u_{3k}u_{3k} & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix} \quad (4)$$

the relationship of the residue to the mode shapes can be clearly seen. When a particular mode is evaluated, every one of the rows and columns of the frequency response matrix can be used to extract that particular mode of the system (providing that the reference is not at the node of a mode). For instance, using the first column of the residue matrix, the mode shape for a particular reference (assuming unit modal mass scaling) can be found from:

$$\begin{Bmatrix} a_{11k} \\ a_{21k} \\ a_{31k} \\ \vdots \end{Bmatrix} = q_k \, u_{1k} \begin{Bmatrix} u_{1k} \\ u_{2k} \\ u_{3k} \\ \vdots \end{Bmatrix} \quad \text{where} \quad q_k = \frac{1}{2j\omega_k} \quad (5)$$

While any row or column can be used, it is very obvious that certain rows or columns (certain references) are better references to select for the generation of good frequency response functions. When directional modes exist in the system, certain references may not be very good for some modes but excellent references of other modes of the system. The modal participation factors help to identify the amount of participation each exciter location provides to the overall excitation of the modes of the system. The frequency response equation can be rewritten in another popular form that identifies the modal participation part of its formulation as:

$$\left[h_{ij}(j\omega)\right] = \sum_{k=1}^{m} \left[\frac{u_{ik} L_{kj}}{(j\omega - p_k)} + \frac{u_{ik}^* L_{kj}}{(j\omega - p_k^*)} \right] \quad (6)$$

From these relationships, the modal participation L of each

Figure 7. First 25 structural modes of the RADARSAT1 satellite.

mode can be clearly seen. While theoretically all the modes can be obtained from any reference location, certain references are much better than other reference locations. Certain references that do not excite the modes well enough will result in measured frequency response functions that may be susceptible to noise and poor dynamic range. These measurements may not be optimum, and use of these measurements for extracting modal parameters is questionable to say the least.

Case Studies

The test geometry of the RADARSAT1 experimental modal test configuration is shown in Figure 2. This structure was tested with five separate shaker excitation locations and 250 measurement points. The main modes of interest for this structure exist in the 10- to 64-Hz frequency band. Several different modal parameter extraction scenarios were performed to show the degradation of the extracted modes when all of the measured degrees of frequency (DOF) are used as opposed to a more selective set of DOF for generating poles and extracting residues. The dataset is then processed using the PolyMAX technique to show the ease with which this difficult data set can be efficiently processed.

Use of *All* Measured Frequency Response Functions. The frequency response measurements were evaluated over eight different bands between 10 and 64 Hz. Poles were extracted using a time-domain, complex, exponential, curve-fitting technique. Typical mode indicator tools were used for identifying modes of the system. The summation function, multivariate mode indicator function and the complex mode indicator function were all used for identifying modes and are shown in Figure 3 for the entire bandwidth. Figure 3 also shows the first stability diagram using the entire bandwidth for evaluation. Clearly, the stability diagram is very difficult to interpret when using the entire bandwidth for all of the references.

Figure 4 shows three separate stability diagrams over three separate bandwidths, where all the references and all measured DOFs are used for the extraction process. The stability diagram was also used for identifying the poles of the system. For this case, the use of all the references and all the measured DOFs were used to extract modal parameter estimates. The mode indicator tools produced adequate identification of the modes of the system, but the stability diagram produces only marginal identification of the poles of the system. Selecting poles from these plots was fairly difficult due to the variance of the estimated pole parameters. Due to the large number of measurements that were obtained from references that did not adequately excite the modes, the stability diagram results are not particularly good. The pole selection adequacy is very questionable.

Once mode shapes were extracted, frequency response functions (FRFs) were synthesized and compared to measured data. Two different synthesized functions are shown in Figure 5. These are not particularly good synthesized comparisons. This is due to the poor extraction of modal parameters from the modal extraction process. These two plots are typical of the synthesized functions for other measurement locations on the structure.

In addition, the modal assurance criteria (MAC) were used for assessing the modes extracted. The matrix plot of the MAC values is shown in Figure 6. The majority of the off diagonal terms are reasonably low, and the extracted data from this perspective appears acceptable even though the synthesized FRFs are not very good. The first 25 modes extracted from the measured data are shown in Figure 7. Many of these modes are

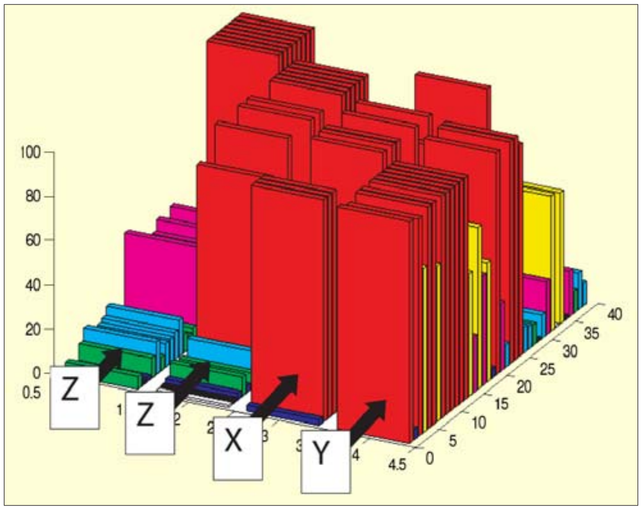

Figure 8. Modal participation matrix of RADARSAT1 satellite.

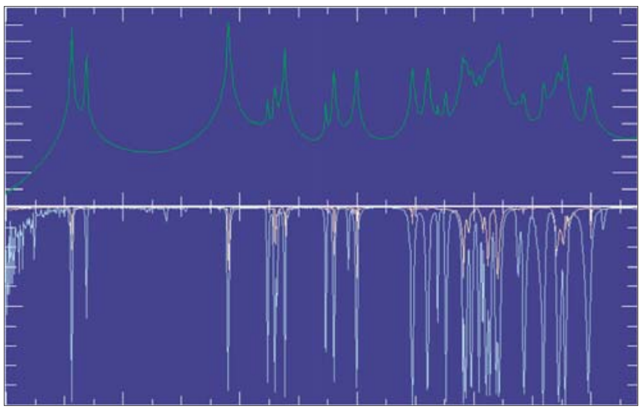

Figure 9. Summation function and mode indicator function using selective references, 10-64 Hz.

primarily local modes of the main radar and solar panels of the satellite. While the modes appear reasonable from mode shape plots and from the MAC, the synthesized FRFs clearly show that the extracted parameters need further scrutiny.

To further evaluate these data, the modal participation factors are plotted in matrix form in Figure 7. The participations seen in Figure 8 clearly show that the first 25 modes are primarily excited by the X-shaker reference location and the y-shaker reference location. The higher frequency modes are activated more significantly from the Z-shaker reference location. To show the detrimental effects of using all the references and all the measured DOFs, a selective set of references and measurement locations was used to determine the modal parameters of the system in the next case study.

Use of Selected Sets of Measured FRFs. For this evaluation, only the X-shaker excitation location and the y-shaker excitation location were used – the Z-shaker excitation locations were not used as references in the evaluation. Again, the frequency response measurements were evaluated over eight different bands between 10 and 64 Hz. Poles were extracted using a time-domain, complex, exponential, curve-fitting technique. Typical mode indicator tools were used for identifying modes of the system. The summation function, multivariate mode indicator function and the complex mode indicator function were all used for identifying modes and are shown in Figure 9 for the entire bandwidth. Comparing Figure 9 with Figure 3 shows that the indicator tools are much easier to interpret.

Figure 10 shows three separate stability diagrams over three separate bandwidths where a selective set of references and a selective set of measurements were used for the evaluation. Comparing Figure 10 to Figure 4 clearly shows the improved situation for selecting poles from the stability diagram. The careful selection of references and measurements for the determination of poles clearly has an impact on the extraction process and improves the selection of poles for the system. The selection of poles is definitely improved through the selective selection of reference location.

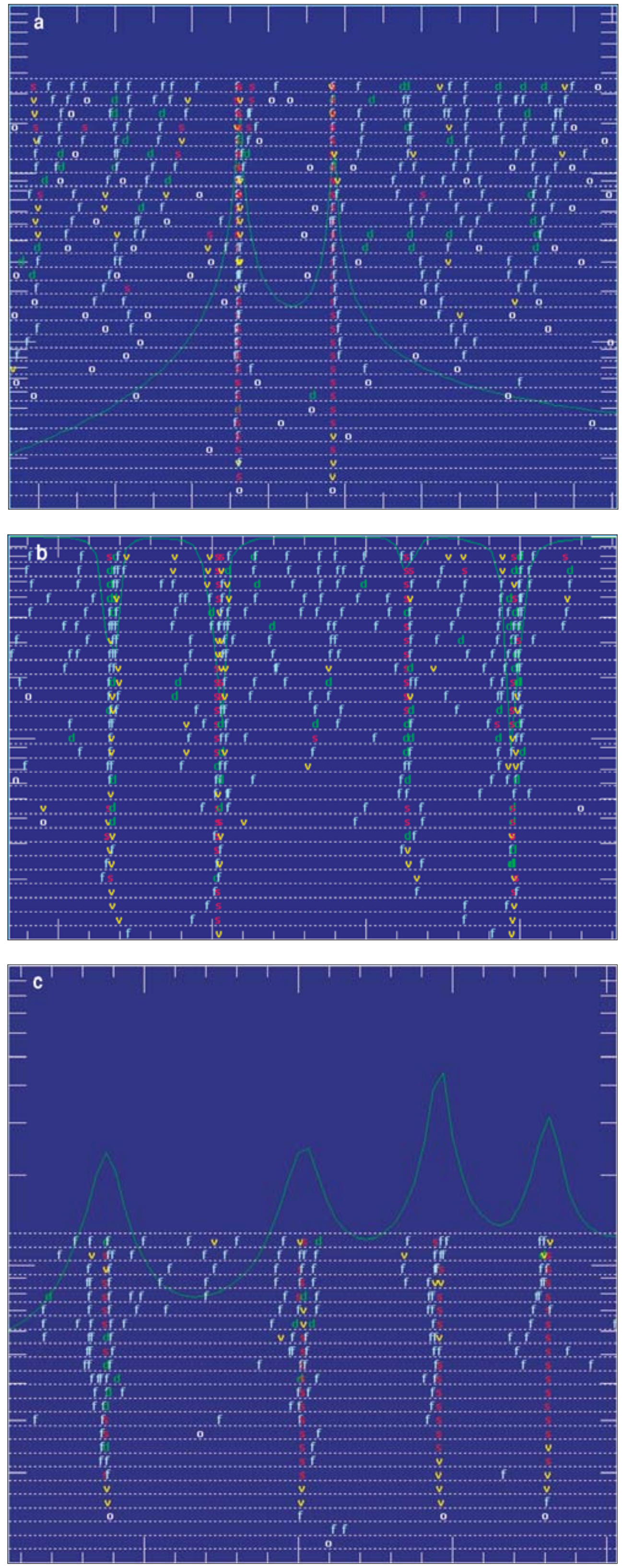

Figure 10. Stability diagram for three different bands using selective references: a) 12.6-20.6 Hz; b) 36.7-40.6 Hz; and c) 44.1-48.1 Hz.

Once the mode shapes were extracted, frequency response functions were synthesized and compared to measured data. Two different synthesized functions are shown in Figure 11. Both of these show very good correlation with the actual mea-

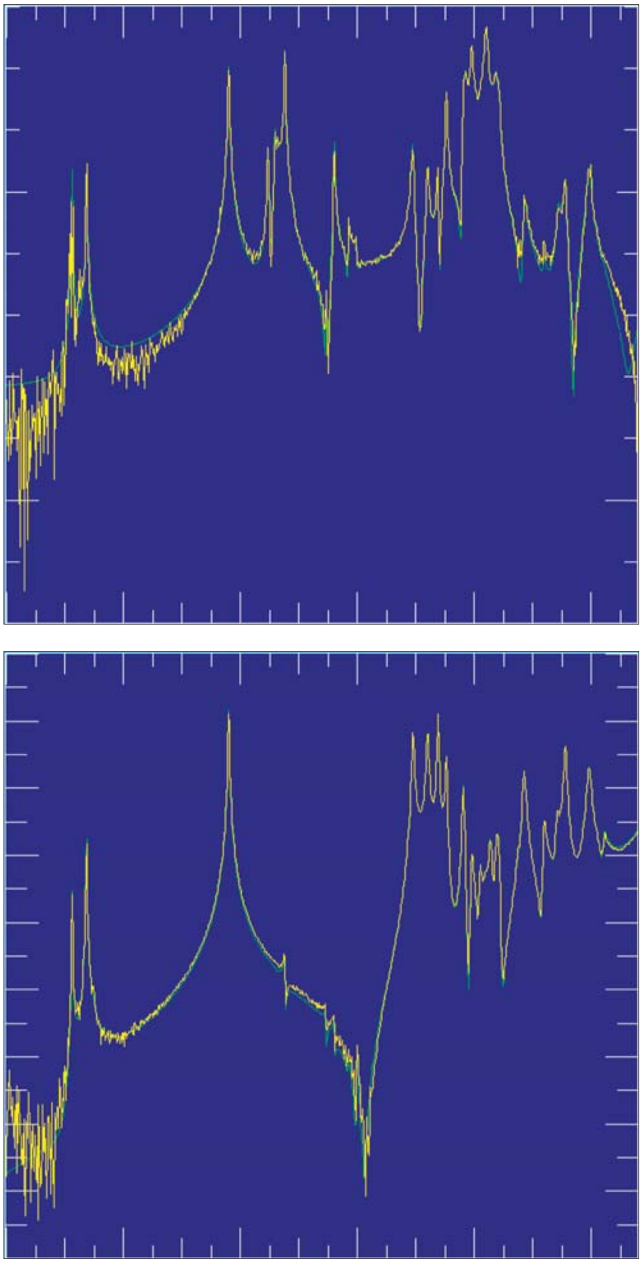

Figure 11. Typical synthesized FRFs using selective references, 10-64 Hz.

sured data and are improved when comparing them to the results shown in Figure 5. These two plots are typical of the synthesized functions for other measurement locations on the structure. Clearly, careful selection of references and measurement locations for extracting modal parameters has a significant effect on the extracted modal parameters.

In addition, the Modal Assurance Criteria (MAC) was used for assessing the modes extracted. The matrix plot of the MAC values is shown in Figure 12. The majority of the off-diagonal terms is reasonably low. Some of the off-diagonal terms may indicate spatial aliasing; additional measurements would minimize this. The MAC is not a particularly good tool for the detailed evaluation of the extracted results. The MAC heavily weights the largest values of the shape and is not a particularly good tool for detailed overall assessment of the extracted parameters. It is shown mainly for reference.

Use of PolyMAX. With the very recent advancement in modal parameter estimation using the PolyMAX approach, wide bands of frequency response measurements can be effectively processed with little restriction on bandwidth and little need to sift the large set of measurements to produce good pole estimates. The PolyMAX approach to modal parameter estimation has revolutionized the modal parameter estimation process. The same data sets described here were reprocessed using all measured DOF for all references. The mode indicator function and complex mode indicator function are shown in Figure 13 for reference. The stability diagram is presented in Figure 14 and is very easy to interpret. Clearly the poles extracted appear to be very well identified over this wide frequency range. Figure 15 compares some synthesized frequency response functions for selective measurements. These synthesized measurements show very good correlation to the actual measurements acquired. In addition, the Modal Assurance Criteria was used for assessing the modes extracted. The matrix plot of the MAC values is shown in Figure 16. The majority of the off-diagonal terms are reasonably low. In comparison to modes extracted using other techniques, the MAC off-diagonal terms are comparable or lower than that of previously extracted mode shapes.

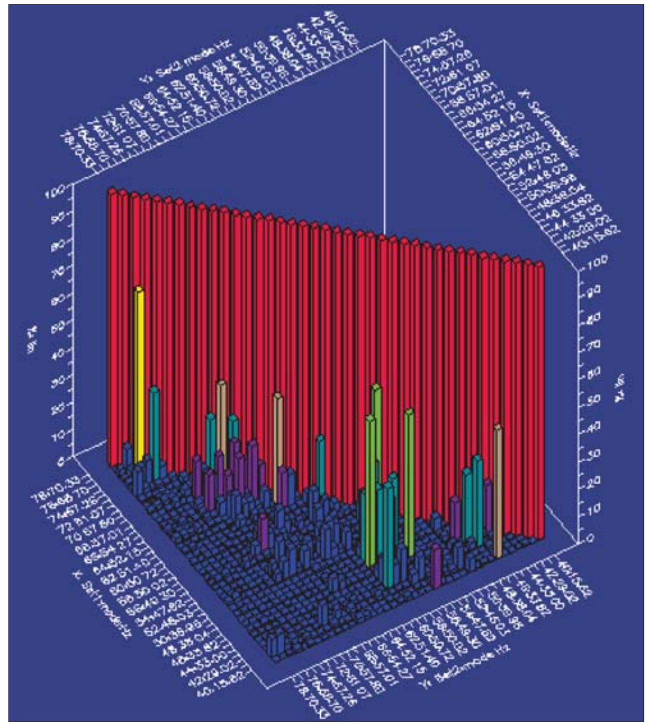

Figure 12. MAC of modes using selective references.

Comparison of PolyMAX and Traditional Techniques

The estimation of parameters from historically used approaches is plagued by noise and mode participation considerations in the estimation process. A significant amount of work is required to sort the data sets into selective bands that are reasonably well excited by the various reference shaker locations to extract acceptable modal parameters. This involves significant time and effort. The newer PolyMAX technique simplifies this process and extracts equivalent parameters using wide bandwidths and without having to selectively sift through the data sets for the best references to excite all modes. PolyMAX is a significant tool for shortening the effort of the reduction of frequency response functions for modal parameter extraction.

Conclusions

Using traditional approaches, extracting modal parameters from multiple-referenced data may need careful selection of measurements used in the process. In general, the use of all measured DOF along with all references may not necessarily produce the best-extracted modal parameters with the techniques historically used. A more selective selection of references and measured DOF for extracting modal parameters is generally required to produce acceptable results for extracted modal parameters with techniques commonly used.

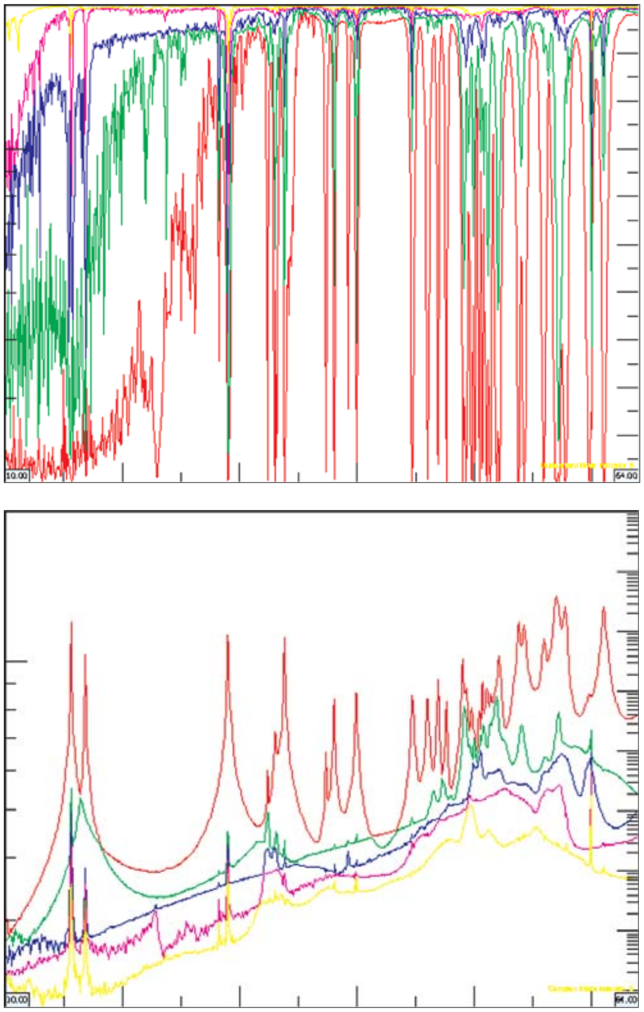

Figure 13. Multivariate mode indicator function (top) and complex mode indicator function using all references (bottom).

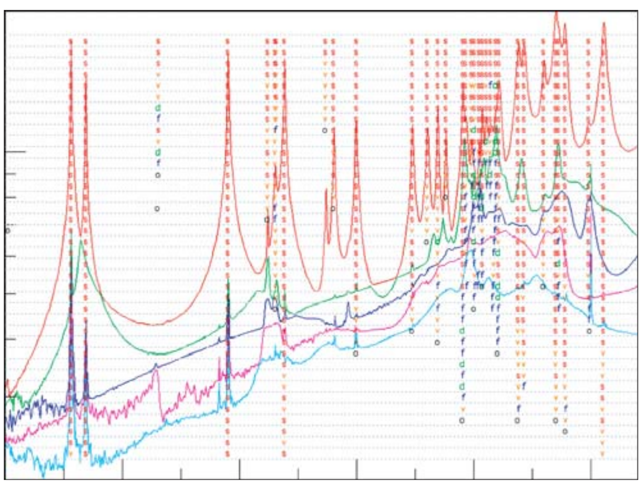

Figure 14. Stability diagram using PolyMAX and all references, 10-64 Hz.

The PolyMAX technique has revolutionized the process of extracting modal parameters. It is shown to be robust and has the ability to extract equal or better modal parameters with significantly less time and user interaction with the pole selection portion of the overall process.

References

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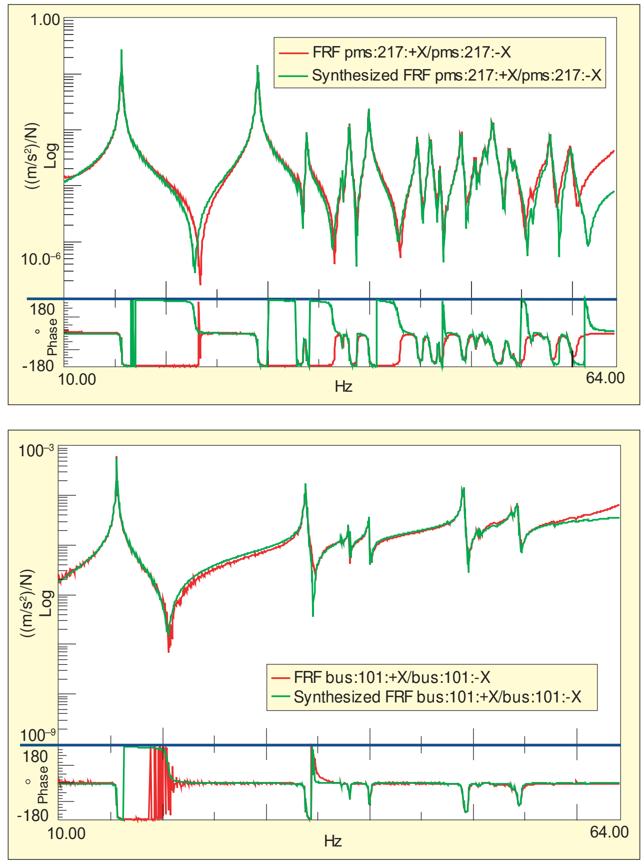

Figure 15. Selective frequency response functions.

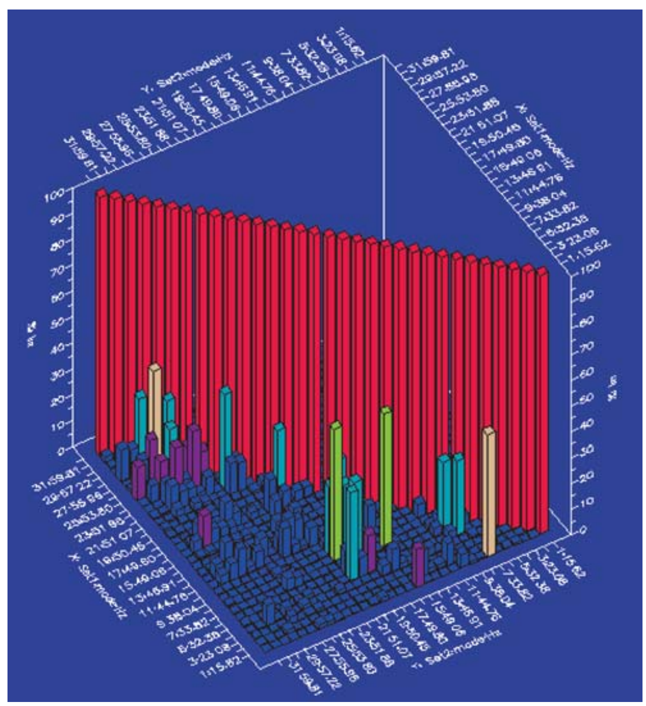

Figure 16. MAC of modes using PolyMAX.

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